McLennan Community College

Hyperbolic functions

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INTRODUCTION

- Hyperbolic functions were introduced in the 1760's independently by Vincenzo Riccati and Johann Heinrich Lambert.
- Hyperbolic functions have similar names to the trigonometric functions but are formed by taking combinations of the two exponential functions e^x and e^{-x}.
- These functions simplify many mathematical expressions and occur frequently in mathematical and engineering applications.
- While ordinary trigonometric functions stem from a circle of unit radius, hyperbolic functions stem from a right half of an equilateral hyperbola.
- In this presentation we will be learning about hyperbolic functions its formulas and graphs, as well as real world applications of these functions.

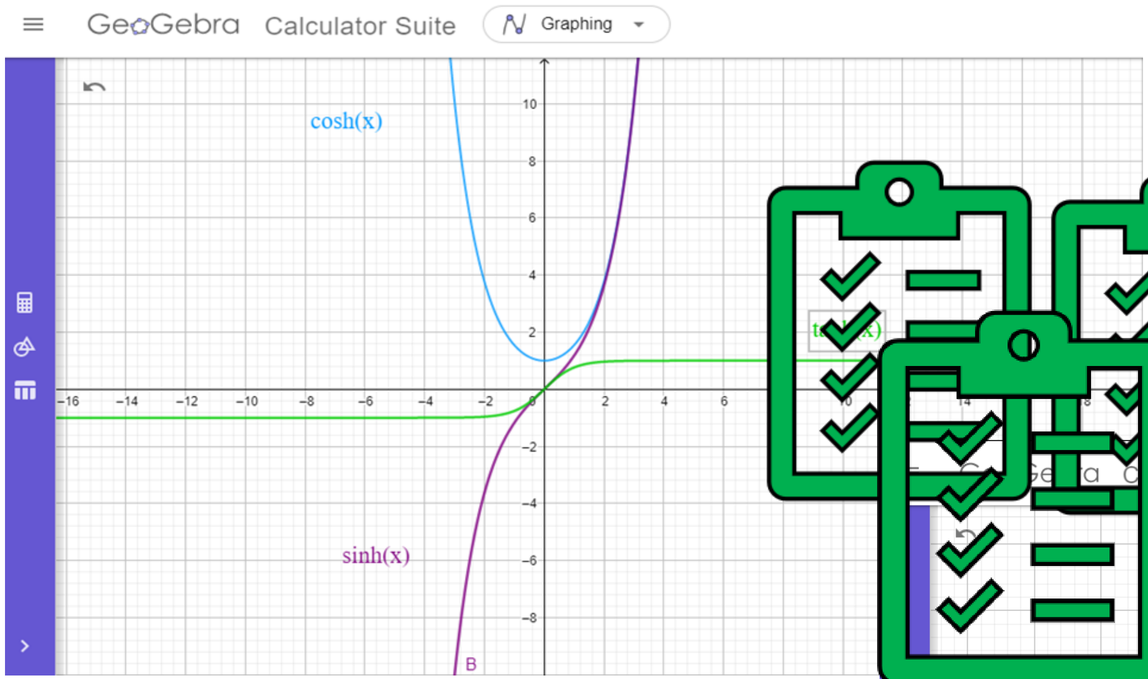
JOHANN HEINRICH LAMBERT

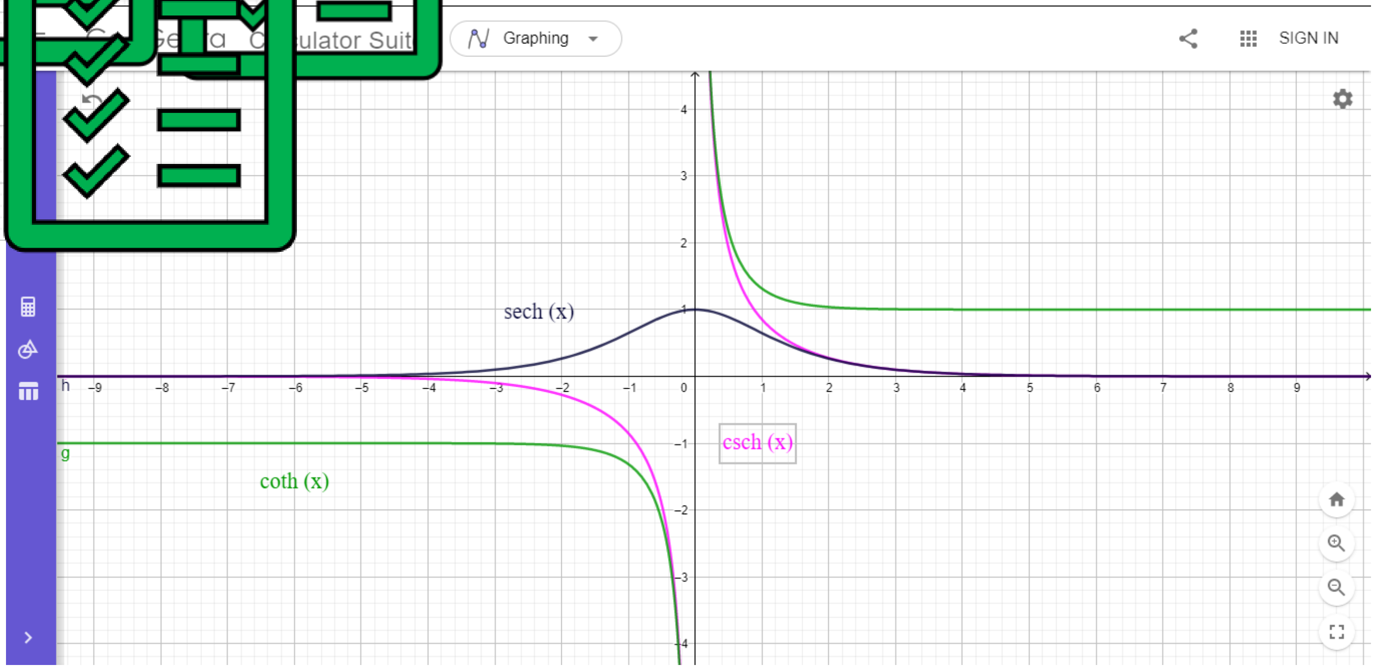
EQUATIONS OF HYPERBOLIC FUNCTIONS

- $\sinh x = \frac{e^x - e^{-x}}{2}$
- $\cosh x = \frac{e^x + e^{-x}}{2}$
- $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

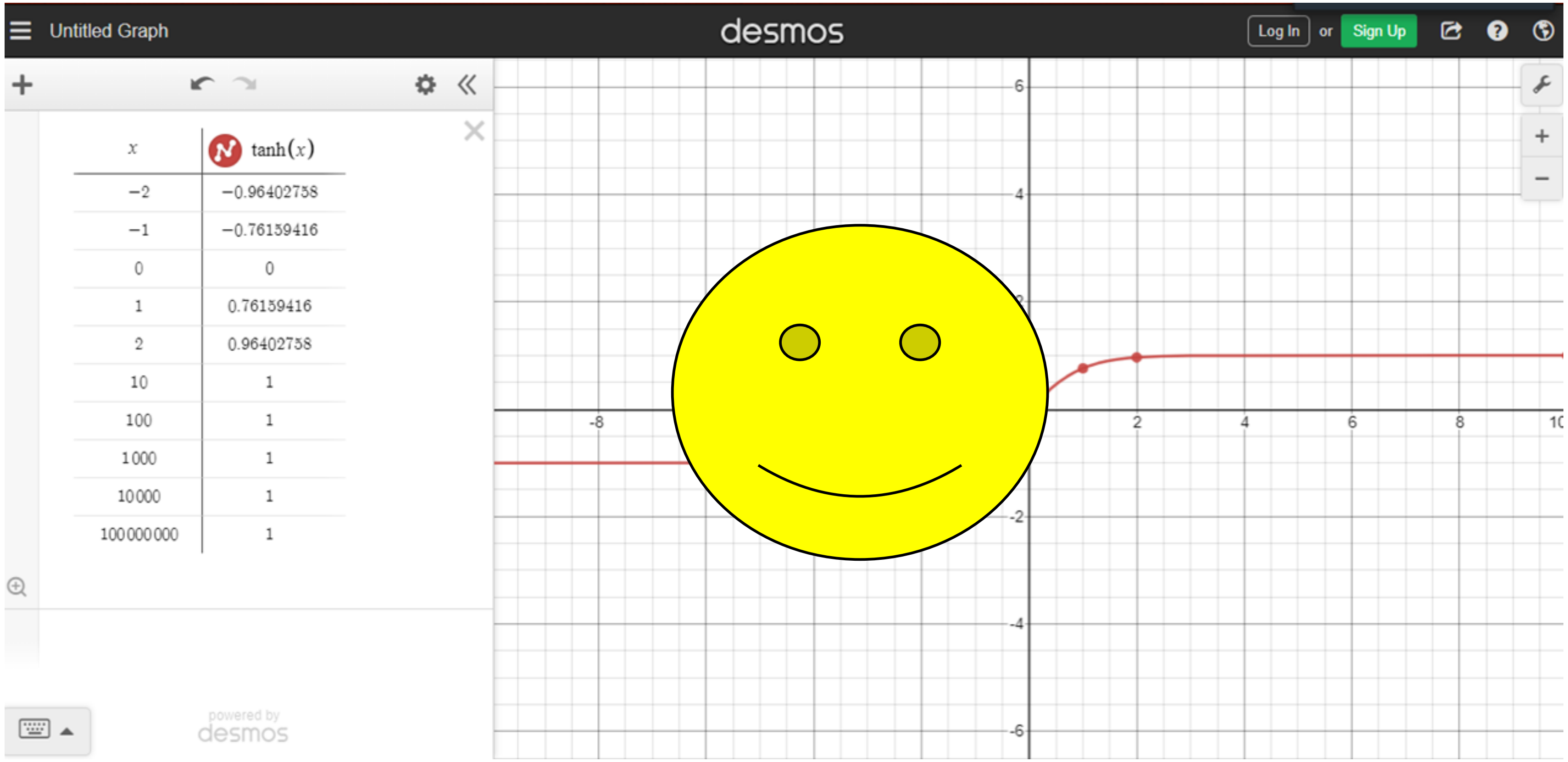
- $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$
- $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$
- $\coth x = \frac{1}{\cosh x} = \frac{\cosh x}{\sinh x} = \frac{\frac{e^x + e^{-x}}{2}}{\frac{e^x - e^{-x}}{2}} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

GRAPHS OF HYPERBOLIC FUNCTIONS





CONSIDERING: $\lim_{x \to \infty} \tanh x$ (graphically)



x	$\tanh(x)$
−2	−0.96402758
−1	−0.76159416
0	0
1	0.76159416
2	0.96402758
10	1
100	1
1000	1
10000	1
100000000	1

CONSIDERING: $\lim_{x\to\infty} \tanh x$ (algebraically)

$$\lim_{x\to\infty} \tanh x$$

$$= \lim_{x\to\infty} \frac{\sinh x}{\cosh x} = \frac{\infty}{\infty}$$

This is an indeterminate form, so we will use L'Hopital's Rule since its infinity over infinity.

L' Hopital's rule states that if you have an equation that has an indeterminate form, then all you need to do is differentiate the numerator and denominator and then take the limit.

CONSIDERING: $\lim_{x\to\infty} \tanh x$ (algebraically)

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$$= \lim_{x\to\infty} \frac{\sinh x}{\cosh x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = \lim_{x\to\infty} \frac{e^x + \frac{1}{e^x}}{e^x - \frac{1}{e^x}}$$

Still $\frac{\infty}{\infty}$

CONSIDERING: $\lim_{x\to\infty} \tanh x$ (algebraically)

$$\lim_{x\to\infty} \tanh x$$

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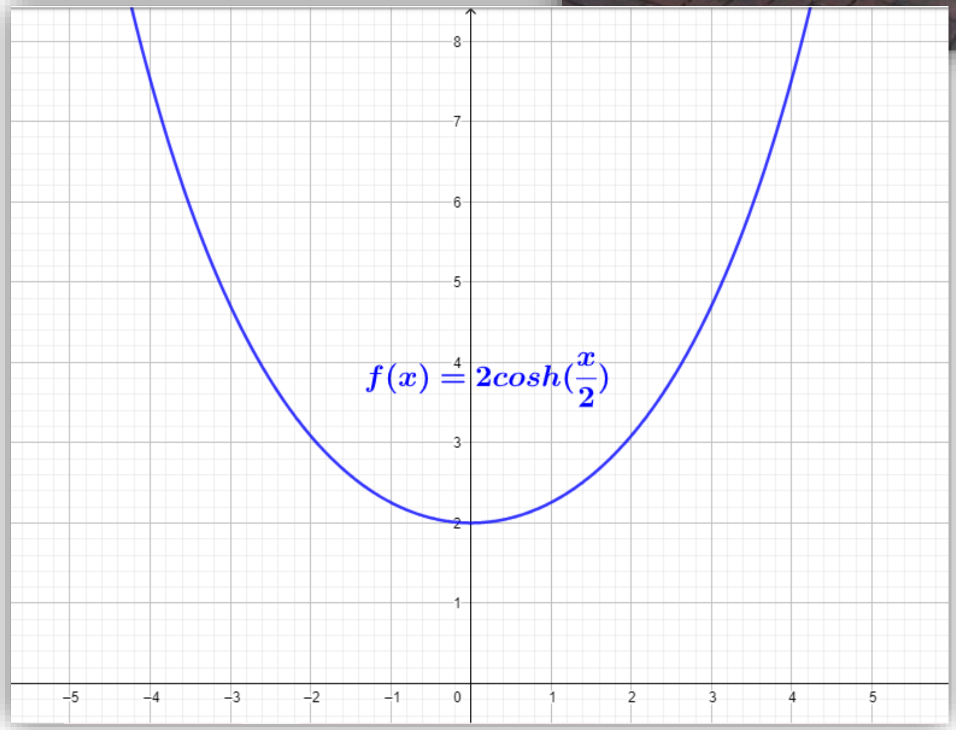
$$= \lim_{x\to\infty} \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}} = \lim_{x\to\infty} \frac{e^{x} + \frac{1}{e^{x}}}{e^{x} - \frac{1}{e^{x}}}$$

$$= \lim_{x\to\infty} \frac{\frac{e^{2x}+1}{e^{x}}}{\frac{e^{2x}-1}{e^{x}}} = \lim_{x\to\infty} \frac{e^{2x}+1}{e^{2x}-1}$$

$$= \lim_{x\to\infty} \frac{2e^{2x}}{2e^{2x}} = 1$$

REAL WORLD APPLICATION

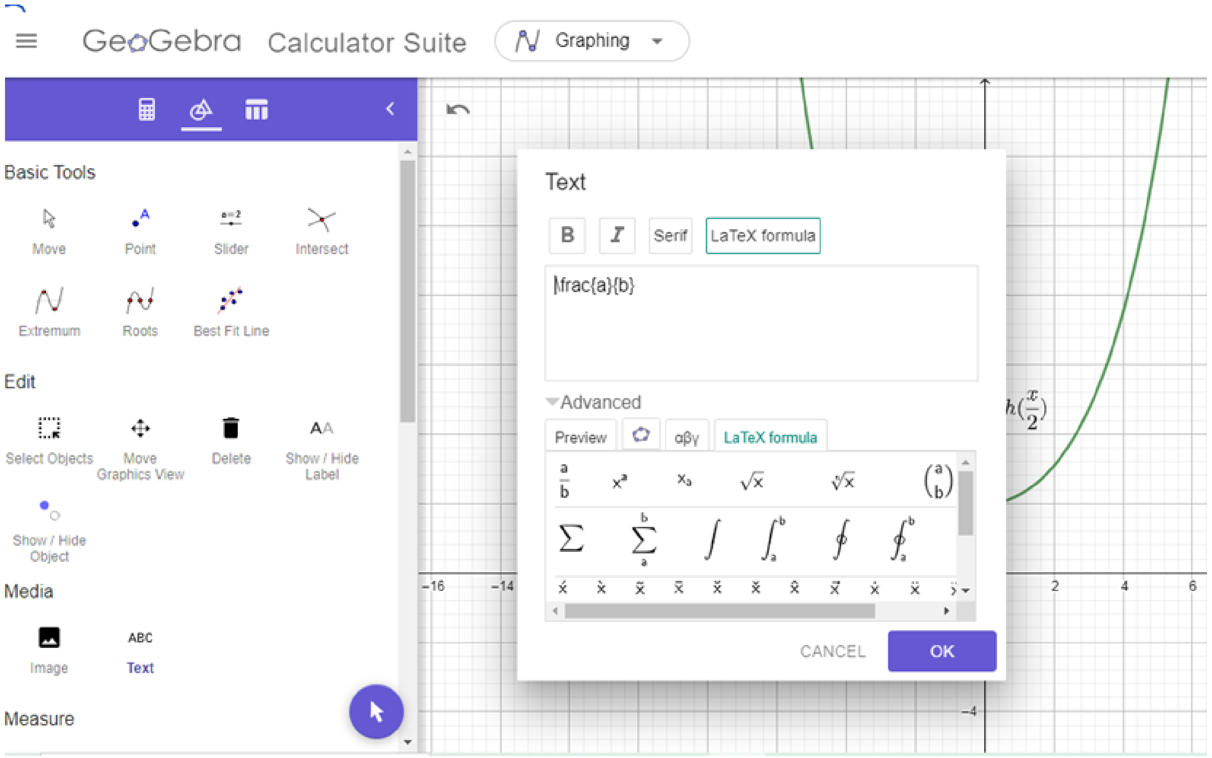
- One physical application of hyperbolic functions involves hanging cables.
- If a cable is suspended between two supports without any load other than its own weight, the cable forms a curve called a ***catenary***.
- High- voltage power lines, chains hanging between two posts, and strands of a spider's web all form catenaries.



Summary

We learned….

- ✓ The formulas and definitions of hyperbolic functions.
- ✓ What their graphs looked like.
- ✓ Who discovered hyperbolic functions.
- ✓ What L'Hoptials rule is
- ✓ How to make your own graphs and label them in GeoGebra.



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