Maximizing Profits in a Business

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The Scenario

Let's say someone starts a business selling calculators. They notice that when they sell them for $40 each, they sell 1200 of them.

They also note that when they price them at $45 each, they only sell 1150.

Each $5 increase in price leads to a decrease of 50 units.

They would like to know at what quantity and price would they make the most profit.

Price, p	$40	$45	$50	$55
Quantity, q	1200	1150	1100	1050

How

In economics, the profit a business entity makes is represented by the function

- $P(x) = r(x) – c(x)$

$r(x)$ = the revenue from selling *x* items

$c(x)$ = the cost of producing *x* items

If $r(x)$ and $c(x)$ are differentiable for *x* in some interval of production possibilities, and if $P(x) = r(x) – c(x)$ has a maximum value there, then it occurs at a critical point of $P(x)$

If it occurs at a critical point, then

$P'(x) = r'(x) – c'(x)$ equals 0 and we see that

- $r'(x) = c'(x)$

In economics, $r'(x) = c'(x)$ means that

At a production level yielding maximum profits, marginal revenue equals marginal cost

Calculating Revenue

Price, p	$40	$45	$50	$55
Quantity, q	1200	1150	1100	1050

Revenue = pq

We can find a slope for this relationship using two points

$$\mathrm{m} = \frac{1150 - 1200}{45 - 40} = -10$$

$$y - y_1 = m(x - x_1)$$

$$\mathrm{q} - 1200 = -10(p - 40)$$

$$q = 1600 - 10p$$

From this we can find p

$$\mathrm{q} = 1600 - 10p$$

$$\frac{-10p}{-10} = \frac{q - 1600}{-10}$$

$$\mathrm{p} = \frac{-1}{10}q + 160$$

Revenue in terms of items is p(q) x q =

$$r(q) = \left(\frac{-1}{10}q + 160\right)q$$

$$r(q) = 160q - \frac{1}{10}q^2$$

Finding cost, MR, & MC

To find cost,

Let's say that one calculator costs $30 to manufacture, plus $2000 a year in fixed costs so

c(q) = 2000 + 30q

P(q) = r(q) – c(q)

$$P(q) = \left(160q - \frac{1}{10}q^2\right) - (2000 + 30q)$$

$$\mathrm{P}(q) = -\frac{1}{10}q^2 + 130q - 2000$$

We can take the derivative of revenue and cost for critical points of the profit function

$$r(q) = 160q - \frac{1}{10}q^2$$

$$r'(q) = 160 - \frac{2}{10}q$$

$$r'(q) = 160 - \frac{1}{5}q = MR$$

$$c(q) = 2000 + 30q$$

$$c'(q) = 30 = \mathrm{MC}$$

Finding a Profit Maximizing Quantity

All we must do next is set MR = MC

$$160 - \frac{1}{5}q = 30$$
$$-\frac{1}{5}q = -130$$
$$q = \frac{-130}{-\frac{1}{5}}$$
$$q = 650$$

This means the profit function has a critical point at 650

The second derivative test lets us know if the critical point is a max or min, depending on the negativity or positivity of the second derivative.

$$\mathrm{P}(q) = -\frac{1}{10}q^2 + 130q - 2000$$
$$P'(q) = -\frac{2}{10}q + 130$$
$$P''(q) = -\frac{1}{5}$$

The function has a maximum value there

Maximized Profits

If we plug in 650 into our equation of P(q)=r(x) – c(x) we can see how much profit the business would make

$$P(q) = \left(160(650) - \frac{1}{10}(650)^2\right) - (2000 + 30(650))$$
$$P(q) = (61{,}750) - (21{,}500)$$
$$P(q) = 40{,}250$$

At a profit-maximizing quantity of 650 units, the business earns $40,250.

If we plug in 651 units into our equation, we get a lower profit.

$$P(q) = \left(160(651) - \frac{1}{10}(651)^2\right) - (2000 + 30(651))$$
$$P(q) = (61{,}779.9) - (21{,}530)$$
$$P(q) = 40{,}249.9$$

Alternative Way to Finding Max Profit

Find vertex of entire P(q) function

Find the derivative and equal it to zero to find this critical point

$$\mathrm{P}(q) = -\frac{1}{10}q^2 + 130q - 2000$$

$$P'(q) = -\frac{2}{10}q + 130$$

$$P'(q) = -\frac{2}{10}q + 130 = 0$$

$$-\frac{2}{10}q = -130$$

$$-\frac{1}{5}q = -130$$

$$-\frac{1}{5}q = -130$$

$$q = -130 * -5$$

$$q = 650$$

Plugging that back into the Profit function gives us the point of maxed profits:

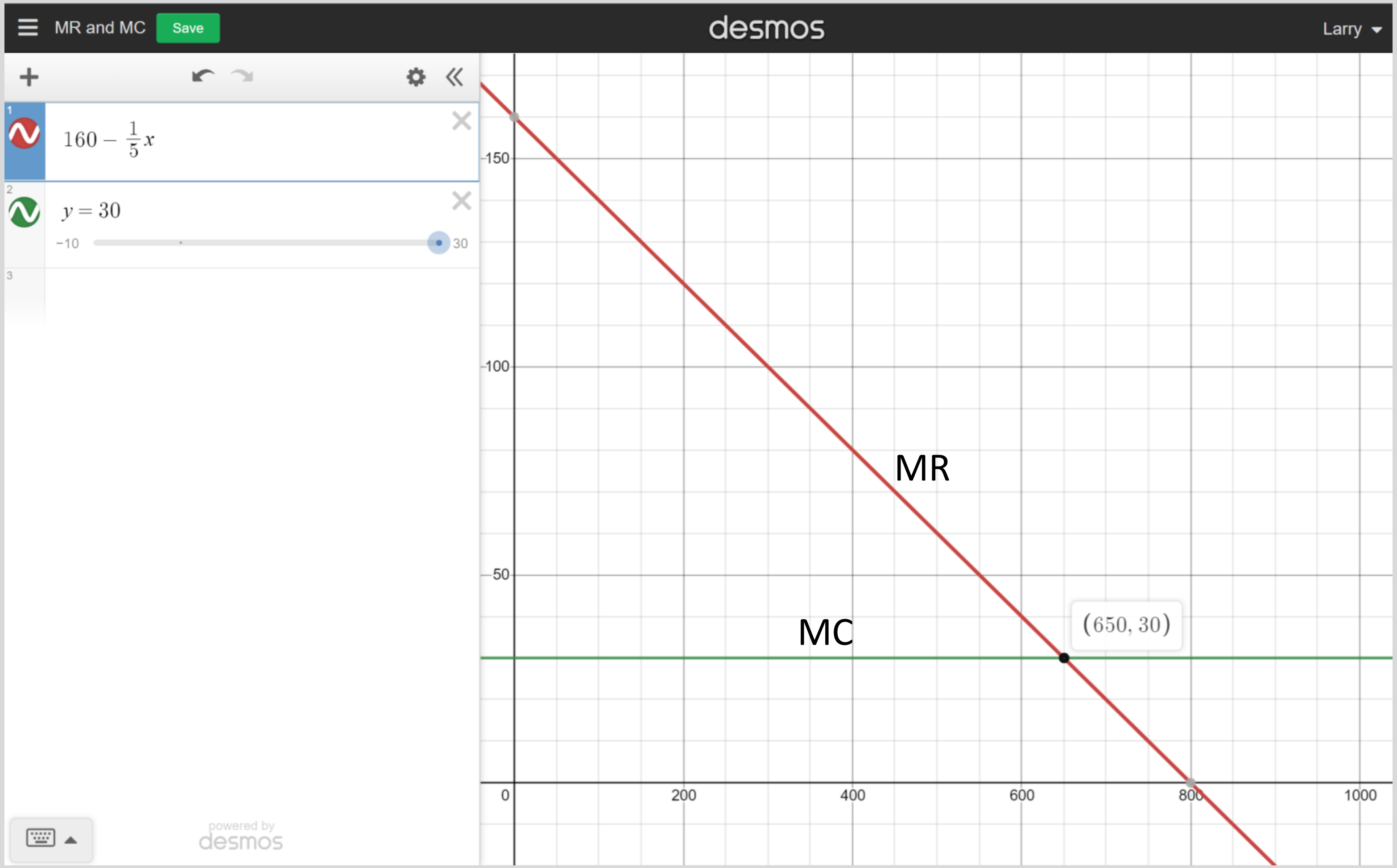
$$P(q) = -\frac{1}{10}(650)^2 + 130(650) - 2000$$

$$P(q) = -\frac{1}{10}(42250) + 84500 - 2000$$

$$P(q) = -42250 + 82500 = 40{,}250$$

(650,40250)

MR = MC



MR and MC
Save
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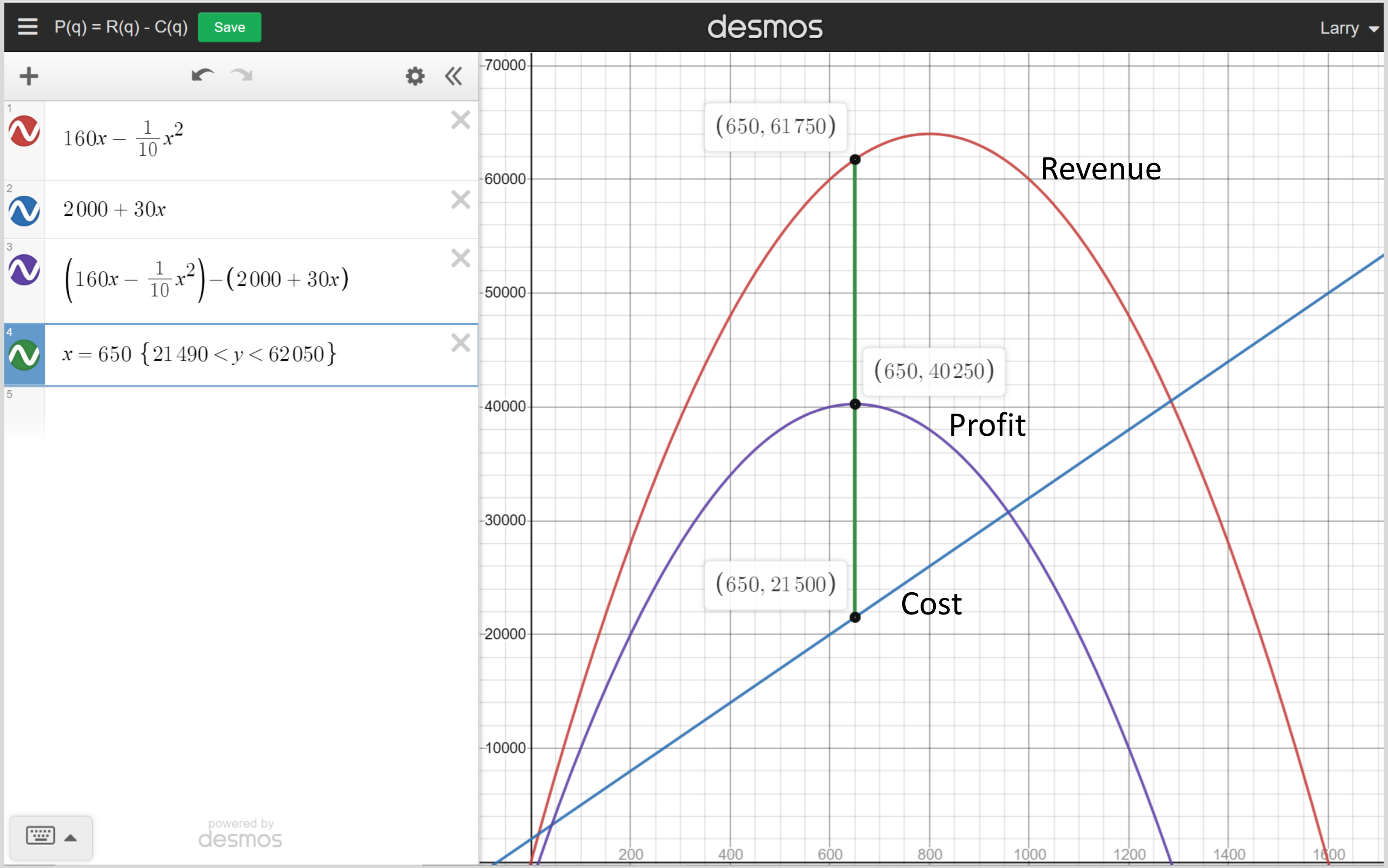
Larry
$160 - \frac{1}{5}x$
$y = 30$
−10
30
150
100
50
0
200
400
600
800
1000
MR
MC
(650, 30)
powered by desmos

Profit Maximizing Quantity & Profit



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