

Exponential change

Taylor Scalf



Exponential functions increase or decrease very rapidly with changes in the independent variable. These changes describe growth or decay in many natural and industrial situations. Such quantities are said to undergo exponential change.

examples:

- Size of a population
- Amount of decaying radioactive material
- A temperature difference between a hot object and its surrounding medium

The law of Exponential change

$$y = y_0 e^{kt}$$

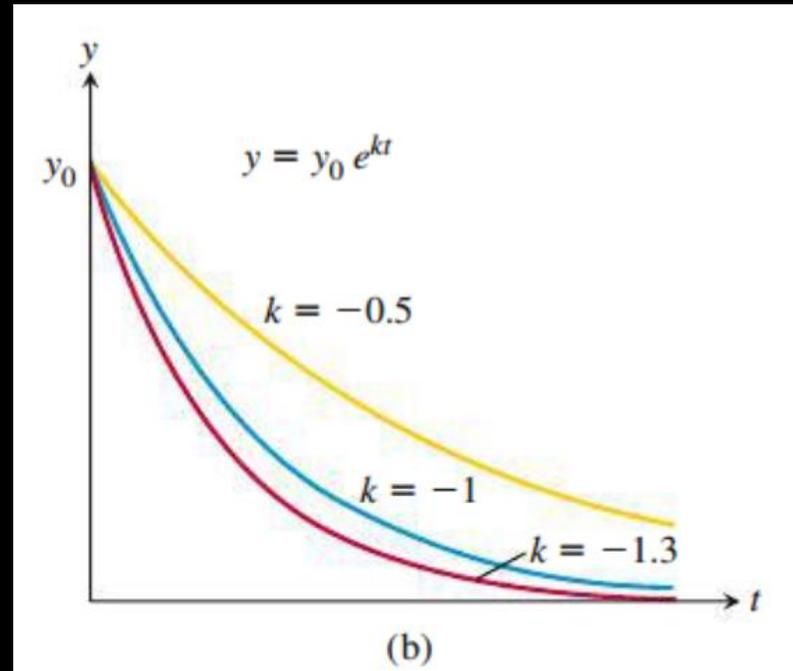
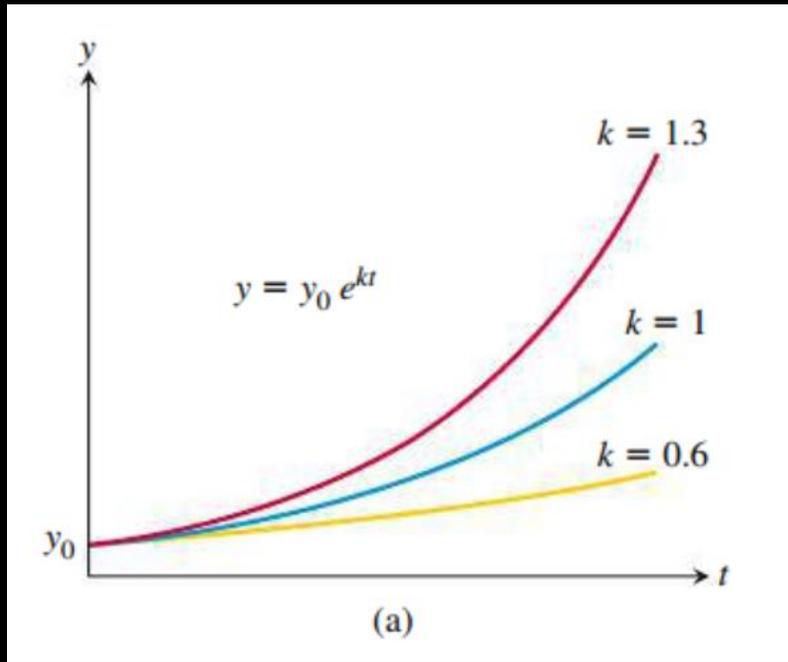
Growth: $k > 0$ Decay: $k < 0$

The number k is the rate constant of the equation.

$$y(t) = y_0 e^{kt}$$

$y(t)$ = current population
 y_0 = initial population
 k = growth rate

Graphs of (a) exponential growth and (b) exponential decay. As $|k|$ increases, the growth ($k > 0$) or decay ($k < 0$) intensifies.



Formulation of the equation:

Differential equation: $\frac{dp}{dt} = kp$

$$\frac{dp}{dt} = kp$$

$$dp = kp dt$$

$$\frac{1}{p} = k dt$$

$$\int \frac{1}{p} dp = \int k dt$$

$$\ln|p| = kt + C$$

$$e^{\ln p} = e^{kt+C}$$

$$p = e^{kt+C}$$

$$e^{kt} * e^C$$

$$p(t) = c * e^{kt}$$

Initial condition when $t = 0$

$$p(t) = c * e^{kt}$$

$$p(0) = c * e^{k(0)}$$

$$p(0) = c(1)$$

$$c = p_0$$

$$p(t) = p_0 e^{kt}$$

$$p(t) = p_0 e^{kt}$$

$p(t)$ = current population

p_0 = initial population

k = growth rate

Real world application

Biologists consider a species of animal or plant to be endangered if it is expected to become extinct within 20 years. If a certain species of wildlife is counted to have 1147 members at the present time and the population has been steadily declining exponentially at an annual rate averaging 39% over the past 7 years, do you think the species is endangered?

$$p(t) = p_0 e^{-kt}$$

$p(t)$ = *current population*

p_0 = *initial population*

k = *growth rate*

$$\begin{aligned} p(t) &= 1147e^{-0.39t} \\ p(20) &= 1147e^{-(0.39)(20)} \\ &= 0.45 \end{aligned}$$

Since the population at $t = 20$ years is less than one ($0.45 < 1$), the species is endangered

Citations

- Thomas' Calculus: Early Transcendentals 14th edition